Palindrome Grab!

In the game of Palindrome Grab, a string of red and black checkers are on the table in a straight line. On your turn, you take checkers from either the left or right end, provided that the collection of checkers you take is a palindrome - in other words, symmetric, same backwards and forwards. Of course one checker is always a palindrome.

For example, if the checkers are RBRRB, from the left you could take R or RBR, and from the right you could take B or BRRB, so you have four choices for your move.

If you don't have checkers, try pennies using heads and tails to represent red and black, for example, or simply write letters on a page and underline the pieces player 1 takes and overline the pieces player 2 takes.

1. Prove that if there are more than 3 checkers on the table, you can always take at least 2 checkers.

2. Can you create a very long string where the most you can take is 2 checkers? If so, describe it. If not, what's the fewest checkers you can possibly have available in a long string?

Greedy Game: The winner is the one who gets the most checkers.

3. In the greedy game, who wins with RBRBBBRRBBBRBRB, player 1 or player 2?

4. Who wins with RRBRRBBRBRRRBRBBRRBRB?

5. Can you explain how to construct a long string where player 1 wins? Can you explain how to construct a long string where player 2 wins?

Patient Game: The winner is the one who gets the *last* checker, regardless of how many each person has collected.

6. Can you explain how to construct a long string where player 1 wins? Can you explain how to construct a long string where player 2 wins?

7. Are there strings where player 1 wins the greedy game but player 2 wins the patient game? How about vice-versa?