Modified by Denise White 2016

Partner Puzzles

The Square Dance

1. Each number from 1 through 16 seeks a partner for a square dance.

   Because it's a square dance, each number when added to its partner must equal a perfect square.

   Is this possible?

   - If not, why not?
   - If so, is there more than one way?

2. What if, you only have numbers 1 through 2?
   Well, that obviously won't work (why not?).
   How about 1 through 4?

3. When is it possible to assign numbers 1 through *n* into pairs for a square dance? When is it impossible? Justify your answers.

Julia Robinson
Mathematics Festival

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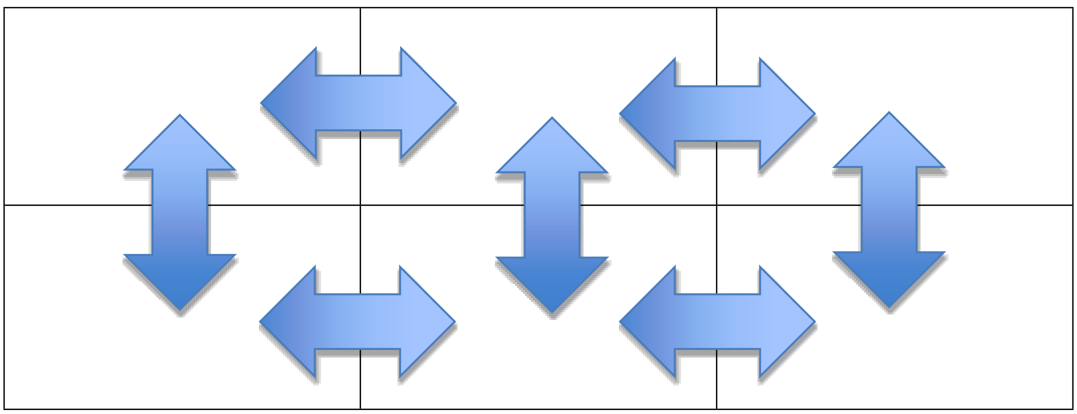
More Dancing

4. The six numbers 1 through 6 are in a dance. They stand in a three by two rectangle such that the seven sums of each vertically or horizontally adjacent pair of numbers are all different. Can you arrange the numbers to make this happen?



5. How many different ways are there to arrange these numbers in their rectangle? (We count rotated or reflected copies of a given arrangement to be the same.)

6. Now the six numbers dance such that, instead of having seven different sums, there are as few different sums as possible. What is the smallest possible number of different sums, and how do you know there can't be fewer?

7. How many ways are there to arrange the numbers 1 through 6 in order to reach this fewest possible number of different sums?

8. Now there are nine numbers, 1 through 9, at the dance. Can they be arranged in a square so that the twelve sums of vertically or horizontally adjacent pairs are all different?

9. Can the numbers 1 through 12 be arranged in a three by four rectangle so that all the sums of horizontally or vertically adjacent pairs are different?

10. Generalize. Can the numbers 1 through mn be arranged in an m by n rectangle so that all the sums of horizontally or vertically adjacent pairs are different?

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Prime Arrangements

11. Arrange the numbers 1 through 16 into the 4 by 4 square. The sum of each vertically or horizontally adjacent pair of numbers must be a prime number (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, or 31).

12. Can you do it without using 31 as one of your sums?

13. How many sums in the grid would you need to check in order to be sure they are all prime?

14. Now repeat the process with a 5x5 square and the numbers 1 through 25.

15. What if all the sums must be at least 11 and at most 41?