Circles on the Road 2011 - Houston

Lesson Plan for the Game of Nim Prepared by Japheth Wood

Description: The mathematical theory of Nim first appeared in print in 1901 (Bouton), and turns out to be the critical example in combinatorial game theory. In fact, the Sprague-Grundy Theorem states that every impartial game is equivalent to a Nim pile. This math circle lesson intends to develop, through the use of selected examples and questions, some key insights into the game of Nim necessary for students to understand the winning strategy for Nim, first proved by Bouton in 1901. Mastery of the game of Nim could be a starting point towards developing a proof of the Sprague-Grundy Theorem as well.

Outline

Part A - Binary Numbers
1. Informally introduce the binary number representation through the Birthday Trick. Five cards are presented, each containing 16 numbers. The mathemagician will guess a volunteer's birth day of the month (a number between 1 and 31) just based on knowledge of which of the five cards include their birth day. This is a classic trick.

Part B - The Game of Nim
2. Present Nim and Jim. Jim is a visual binary representation of Nim, but this won't be obvious to many.
A Nim game consists of several piles of counters. Players alternate moves, and a valid move consists of choosing a pile, and taking 1 or more counters from that pile. The player to take the last counter wins. Equivalently, the first player who can't make a valid move has just lost. (Note that in some versions of Nim, the player who is forced to take the last counter loses. This variation is called "misere", and will not be presented in this session.)
The other game is Jim (short for Japheth's Nim) consists of several rows of red and yellow counters. Players alternate moves, and a valid move consists of picking a row and changing the color (red to yellow or yellow to red) of one or more counters. The leftmost counter changed must be from yellow to red. The first player who can't make a valid move (no more yellow counters remain) loses.
The game of Jim is exactly a visual representation of Nim in binary. Each row, when read in binary (yellow = 1 and red = 0) is the number of counters in the corresponding Nim pile. This equivalence, while perhaps not immediately obvious, will be made clear to the participants after the strategy for Jim has been found.
3. Develop strategies and understandings for both Nim and Jim. Through experience and analysis of 2 pile Nim and 2 row Jim games, participants will learn the basic concepts of game theory, including what a winning strategy is, the game tree, N and P games, and the copy-cat strategy. A game consisting of two identical Nim piles or Jim rows is a P game, and a game consisting of two different Nim piles or Jim rows is an N game.
A generalization of the copy-cat strategy gives an elegant matching strategy for three-rowed Jim. This visual strategy is then translated through binary numbers into the well-known strategy for Nim.

Part C - The Sprague-Grundy Theorem
Every impartial game is equivalent to a Nim pile.
4. The concept of adding games and equivalent games is introduced through examples.
5. The concept of a game as the set of options is introduced.
6. The Sprague-Grundy Theorem develops naturally out of examples and reasoning.

In the interest of time, we will plan parts A and B, and leave part C for another occasion.

References:

Elwyn R. Berlekamp John H. Conway and Richard K. Guy, Winning Ways for your mathematical plays, Academic Press, London, 1982.

Charles L. Bouton, Nim, A Game with a Complete Mathematical Theory, The Annals of Mathematics Vol. 3, No. 1/4 (1901 to 1902), 35-39.